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A simple theory of capillary-gravity wave turbulence

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Employing a recently proposed ‘multi-wave interaction’ theory (Glazman 1992), inertial spectra of capillary-gravity waves are derived. This case is characterized by a rather high degree of nonlinearity and a complicated dispersion law. The absence of scale invariance makes this and some other problems of wave turbulence (e.g. nonlinear inertia-gravity waves) intractable by small-perturbation techniques, even in the weak-turbulence limit. The analytical solution obtained in the present work for an arbitrary degree of nonlinearity is shown to be in reasonable agreement with experimental data. The theory explains the dependence of the wave spectrum on wind input and describes the accelerated roll-off of the spectral density function in the narrow sub-range separating scale-invariant regimes of purely gravity and capillary waves, while the appropriate (long- and short-wave) limits yield power laws corresponding to the Zakharov-Filonenko and Phillips spectra.

1. Introduction

The subject of this paper is turbulence of surface gravity-capillary waves, although the formalism can be applied to other problems of nonlinear wave dynamics, such as inertia-gravity and Rossby waves in the ocean and atmosphere, etc. The weak turbulence theory presently available for these problems (Zakharov, L’vov & Falkovich 1992) proved successful in many cases. However, some of its constraints considerably limit its scope. In particular, weak turbulence theory requires scale invariance (as yielded by a power-law type of dispersion law) and localization of external sources and sinks in the wavenumber-frequency space to yield practical results even for weakly nonlinear problems. Owing to formidable mathematical difficulties, weak turbulence theory does not account for higher-order nonlinear effects. Therefore, some intuitive and less formal approaches may prove advantageous in many cases. An example is given by Phillips (1985) where weak turbulence of deep-water surface gravity waves is considered with the source functions continuously distributed in the wavenumber space. A similar approach, but going beyond the weak turbulence limit (and called the ‘multi-wave interaction theory’) was suggested recently (Glazman 1992) to explain observed variations in the exponent for power-law spectra of surface gravity waves. The Kolmogorov assumption of locality of nonlinear wave-wave interactions is crucial in these theories. Provided this assumption remains approximately valid for an increased number of the resonant Fourier components, multi-wave interaction theory could in principle be applied to a broad class of problems. Indeed, it does not require the lowest degree of nonlinearity, simple dispersion laws or simple expressions for the wave energy density, and it can be used for weakly non conservative systems – as

demonstrated earlier. However, owing to its heuristic nature, multi-wave interaction theory needs thorough experimental verification.

Capillary-gravity waves are characterized by a highly complex expression for the potential energy,

$$U = \frac{1}{2} \rho g \int \eta^2 dx + \sigma \rho \int [(1 + |\nabla \eta|^2)^{1/2} - 1] dx, \quad (1.1)$$

where $\eta = \eta(x, t)$ is the elevation of the fluid surface above the zero-mean level, g is the acceleration due to gravity, and σ is the coefficient of surface tension divided by fluid density ρ . The dispersion law is

$$\omega^2 = gk + \sigma k^3, \quad (1.2)$$

which, while permitting three-wave resonant interactions, eliminates scale-invariance (actually, statistical self-affinity) of the wave field. Characteristic wavelengths at which (1.1) and (1.2) are relevant extend from hundredths and up to ten centimetres – the waves amenable to accurate laboratory investigation. Thus, the capillary-gravity waves are highly interesting as a test case. Besides, these waves are primarily responsible for radar backscatter by the ocean surface, and thus are of great practical interest.

Laboratory (Jähne & Riemer 1990) and field (Hwang *et al.* 1993; Hara, Bock & Lyzenga 1994) observations showed that, contrary to intuitive expectations, the wavenumber spectrum in the capillary-gravity range does not exhibit a monotonic transition from the gravity wave to the capillary wave regime. Instead, it experiences an accelerated roll-off at a rate exceeding the rates of both gravity and capillary spectra roll-offs. The accelerated roll-off commences at wavenumbers which show no noticeable dependence on the wind energy input (Jähne & Riemer 1990), although the influence of long waves appears to be important (Hara *et al.* 1994). This behaviour finds a simple explanation in the framework of the present theory.

In §2, the theoretical approach is reviewed. Spectra of capillary-gravity waves are derived in §3. In its present form, the theory ignores some interesting, although well-known, effects of longer waves on the capillary-gravity ripples (e.g. Phillips 1981). These include a highly non-local (in the wavenumber space) energy exchange through the radiation stress exerted by longer waves and through the generation of ‘parasitic capillaries’ at the crests of steep gravity waves (Longuet-Higgins 1963). Other factors of short-long wave interactions include an additional acceleration ‘felt’ by the short-scale waves riding on top of longer waves. Since the present study focuses only on the energy transfer by the local inertial cascade, some discrepancies with the observations are to be expected; these are discussed in §4.

2. Multi-wave interaction theory for capillary-gravity waves

Let us consider a conservative spectral flux of wave energy. The external energy source acts at lower frequencies–outside our inertial range. Therefore, a specific mechanism of wave generation is not addressed here. The rate Q of energy input, assumed to be known, equals the rate of energy transfer down the spectrum. Following the earlier reasoning (Glazman 1992), Q is related to the characteristic time of nonlinear wave-wave interaction (the ‘turnover time’), t_n , and the characteristic energy E_n transferred from a cascade step n to step $(n+1)$ by

$$\rho Q = E_n / t_n, \quad (2.1)$$

where the water density ρ appears because Q is taken per unit mass of water. Provided E_n and t_n can be expressed in terms of k , ω and wave amplitude a , equation (2.1) allows

one to derive the spectrum by means of elementary algebra (e.g. Frisch, Sulem & Nelkin 1978; Larraza, Garrett & Putterman 1990). Let us express these parameters in terms of the relevant quantities.

An approximate equi-partition of energy between the kinetic and the potential parts allows one to write the surface density of the total wave energy, E , as

$$E \approx \rho[g\langle\eta^2\rangle + \sigma\langle(\nabla\eta)^2\rangle], \quad (2.2)$$

where the angular brackets denote an ensemble average. To pass from (1.1) to (2.2) we assumed $\langle(\nabla\eta)^2\rangle \ll 1$ which is justified for natural seas (e.g. Cox & Munk 1954). The energy E is related to the spectral density of the wave energy by

$$E = \int S(\omega) d\omega = \iint F(k, \theta) k d\theta dk, \quad (2.3)$$

where the integration is carried out over all wavenumbers and frequencies. Here, $S(\omega)$ is the frequency spectrum and $F(k, \theta)$ is the two-dimensional wavenumber spectrum of the wave energy.

The amount of energy, E_n , transferred by the cascade mechanism during time t_n is estimated as

$$E_n = \int_{\omega_n}^{\omega_{n+1}} S(\omega) d\omega, \quad (2.4)$$

where (ω_n, ω_{n+1}) is the width of a cascade step (which must be much smaller than the width of the inertial range), and the ratio $r = \omega_{n+1}/\omega_n$ is constant and sufficiently greater than unity—as required by the assumption of locality of wave-wave interactions in the frequency space. Indeed, differentiating (2.4) over ω_n yields

$$-dE_n/d\omega_n = S(\omega_n) - rS(r\omega_n) = S(\omega_n)[1 - r^{1-p}],$$

where the latter equality is valid for wave spectra of type $S(\omega) \propto \omega^{-p}$. Provided the spectrum rolls off sufficiently fast (i.e. $r^{1-p} \ll 1$), we have

$$S(\omega) \approx -dE(\omega)/d\omega. \quad (2.5)$$

Although the spectrum being derived does not follow a power law ω^{-p} , the above approximation can be easily checked *a posteriori*. From (2.2) it follows that E_n for gravity-capillary waves can be written as

$$E_n \approx \rho[ga_n^2 + \sigma(a_n k_n)^2]. \quad (2.6)$$

Here, a_n is the Fourier amplitude of surface oscillation at the frequency/wavenumber scales ω_n and k_n , corresponding to the n th step in the spectral cascade.

The derivation of the turnover time is formally based on the scaling of the collision integral in the kinetic equation (Zakharov & L'vov 1975; Phillips 1985; Larraza *et al.* 1990). However, we shall introduce this timescale in a less formal fashion which leads to useful generalizations. To this end let us notice that the nonlinearity of the wave process is measured by the ratio, ϵ , of the fluid particle velocity, u , to the wave phase velocity, $c = \omega/k$ (Whitham 1974). Since fluid particles in a surface wave on deep water execute an approximately orbital motion in the vertical plane with radius equal to the wave amplitude and period $2\pi/\omega$, the value of u at a given scale is estimated as $a_n \omega_n$. Correspondingly, the ratio u/c is

$$\epsilon_n \equiv \frac{u_n}{\omega_n/k_n} \approx \frac{a_n \omega_n}{\omega_n/k_n} = a_n k_n. \quad (2.7)$$

This quantity represents the small parameter in deterministic perturbation theories. However, since the kinetic equation for the wave action, $N(\mathbf{k}) = F(\mathbf{k})/\omega$, is derived for second statistical moments, the equations of statistical theory are developed in powers of ϵ^2 . Terms (i.e. collision integrals) of order ϵ^2 correspond to three-wave interactions, while each additional Fourier component accounted for in the interaction integral adds new terms which are ϵ^2 times as great as a preceding term. The ν th term is of order $\epsilon^{2(\nu-1)}$. Thus, the characteristic time of nonlinear wave-wave interactions increases as the number of interacting harmonics grows. For three-wave interactions, this time is given by $t^{-1} \approx \omega\epsilon^2$, and for an arbitrary number, ν , we have (see also Larraza *et al.* 1990)

$$t^{-1} \approx \omega\epsilon^{2(\nu-2)}. \quad (2.8)$$

Relationship (2.8) is very convenient, for it allows one to carry out all calculations in a general form and then take appropriate limits for long- and short-wave asymptotics: $\nu = 4$ for gravity waves on deep water and $\nu = 3$ for capillary waves.

Apparently, the actual number of the resonantly interacting wave harmonics in the transitional, gravity-capillary range should be allowed to take values between 3 and 4. This leads us to view ν as a statistical quantity. Using this broader interpretation of ν , it is then natural to further assume that the mean 'effective' number of the resonantly interacting harmonics should increase with an increasing degree of the wave nonlinearity (Glazman 1992). The following heuristic argument hopefully makes this point more transparent.

In the absence of ambient fields (such as variable currents or long-wave oscillations), the kinetic equation is given by

$$\partial N/\partial t + \nabla_{\mathbf{k}} \cdot T(\mathbf{k}) = p(\mathbf{k}), \quad (2.9)$$

where $p(\mathbf{k})$ is the spectral density of the input flux of wave action (from wind), and $\nabla_{\mathbf{k}} \cdot T(\mathbf{k})$ denotes the divergence of the action flux in the wavenumber space. In a random wave field, a few waves whose steepness is well above the average can always be found. Hence, the degree of wave nonlinearity may be locally very high. Accounting for the corresponding higher-order terms in the kinetic equation (derived for the averaged quantities), one can formally write

$$\nabla_{\mathbf{k}} \cdot T(\mathbf{k}) = I_3 + I_4 + \dots + I_m + \dots \quad (2.10)$$

Here, I_m are collision integrals accounting for interactions among m waves satisfying resonance conditions

$$\omega_0 \pm \omega_1 \pm \dots \pm \omega_m = 0, \quad \mathbf{k}_0 \pm \mathbf{k}_1 \pm \dots \pm \mathbf{k}_m = 0 \quad (2.11)$$

(non-resonant terms can be eliminated by appropriate canonical transformations (Zakharov *et al.* 1992). It has been argued (Glazman 1992) that intermittently occurring rare events of steep and breaking waves (characterized by a locally high nonlinearity, hence a large, or even infinite, number of interacting Fourier components), result in an increased mean (over a large time interval and large surface area) number ν of the resonantly interacting harmonics. While this ν may be substantially greater than the minimum resonant number appearing in weak turbulence theory, the energy and action spectral transfer may still be dominated by the weakly nonlinear inertial cascade. Thus, the 'effective' ν is introduced as an unknown function of the problem, the assumption of locality of wave-wave interactions in the wavenumber space remaining in force. Although the total flux of the wave action is

composed of many partial fluxes, the turnover time given by (2.8) is determined by the slower components of (2.10). In the absence of a rigorous theory for the effective value of ν in (2.8), we shall relate this quantity to external parameters in a heuristic way in §4.

Let us consider the case of an external input concentrated at wavenumbers below a certain k_0 marking the high-wavenumber boundary of the ‘generation range’. That is, at $k > k_0$, $p(k) = 0$, and the spectral flux is purely inertial. It is given by

$$\rho Q = \int_0^{k_0} \omega(k) k dk \int_{-\pi}^{\pi} p(k, \theta) d\theta. \quad (2.12)$$

Correspondingly, equation (2.9) for the inertial range yields

$$E_n \omega_n (a_n k_n)^{2(\nu-2)} \approx \rho Q (= \text{const}), \quad (2.13)$$

where $n \geq 1$.

Using (1.2) and (2.6), equation (2.13) results in

$$E_n \approx \rho Q^{1/(\nu-1)} \sigma^{(\nu-2)/(\nu-1)} \omega_n^{-1/(\nu-1)} \Phi_\nu(\omega_n), \quad (2.14)$$

where

$$\Phi_\nu(\omega_n) = \left[\frac{1 + M(\omega_n)}{M(\omega_n)} \right]^{(\nu-2)/(\nu-1)}, \quad M(\omega) = [k(\omega)/\kappa]^2 \quad (2.15)$$

and $1/\kappa = (\sigma/g)^{1/2}$ gives the characteristic lengthscale of the problem. The explicit dependence of k on ω , as follows from (1.2), is

$$k(\omega) = u_1(\omega) + u_2(\omega), \quad (2.16)$$

where

$$u_{1,2}(\omega) = \left[\frac{\omega^2}{2\sigma} \pm (D(\omega))^{1/2} \right]^{1/3} \quad \text{and} \quad D(\omega) = \frac{4g^3 + 27\omega^4\sigma}{108\sigma^3}. \quad (2.17)$$

Based on (2.14) and (2.5), the energy spectrum is found as

$$S(\omega) = \alpha \rho \sigma \left(\frac{Q}{\sigma} \right)^{1/(\nu-1)} \frac{\Phi_\nu(\omega)}{\nu-1} \left[1 + \frac{4(\nu-2)}{1+3M(\omega)} \right] \omega^{-\nu/(\nu-1)}, \quad (2.18)$$

where α is a (‘Kolmogorov’) constant of proportionality. The short-wave limit of (2.18) is obtained by setting $M(\omega) \rightarrow \infty$, hence $\Phi_\nu(\omega) \rightarrow 1$. The long-wave limit is found by setting $M(\omega) \ll 1$, hence $\Phi_\nu(\omega) \approx (M(\omega))^{-(\nu-2)/(\nu-1)}$.

3. Wave spectra

For capillary-gravity waves, relationships between the energy spectrum (2.18) and the spectra of surface height and surface gradient (i.e. wave slope) are more complicated than the corresponding relationships for pure gravity and pure capillary waves. Specifically, as follows from (2.2), the spectrum of surface height variation is related to (2.18) by

$$S_\eta(\omega) = \frac{S(\omega)}{\rho g [1 + M(\omega)]}. \quad (3.1)$$

In the special case of short waves and $\nu = 3$, this yields the Zakharov–Filonenko spectrum (Zakharov & Filonenko 1967) of weakly nonlinear capillary waves. It is also easy to check that the long-wave limit of (3.1) yields spectra of surface gravity waves: the Zakharov–Filonenko spectrum (Zakharov & Filonenko 1966) for $\nu = 4$ and the Phillips spectrum for $\nu \rightarrow \infty$.

In the wavenumber domain, the two-dimensional spectrum of surface height variation (omitting the directional factor, $\Psi(\theta, k)$) is found as

$$F_\eta(k) = k^{-1} \left[S_\eta(\omega) \frac{d\omega}{dk} \right]_{\omega=\omega(k)}. \quad (3.2)$$

For simplicity, we assume the following normalization condition for $\Psi(\theta, k)$:

$$\int_{-\pi}^{\pi} \Psi(\theta, k) d\theta = 1. \quad (3.3)$$

The two-dimensional spectrum of the wave slope modulus (again, the directional factor $\Psi(\theta, k)$ is omitted) is found as

$$F_{\nabla\eta}(k) = k^2 F_\eta(k) = \frac{(kg)^{1/2} [1 + 3M(k)]}{2\rho g [1 + M(k)]^{3/2}} S(\omega(k)). \quad (3.4)$$

It is useful to present these results in a non-dimensional form by scaling all variables as follows:

$$k = K\kappa, \quad \omega = \Omega\tilde{\omega}, \quad \tilde{Q} = \frac{Q}{(\tilde{\omega}/\kappa)^3}, \quad \tilde{S}(\Omega) = \frac{S(\omega)\kappa^3}{\alpha\rho\tilde{\omega}}, \quad (3.5)$$

where $\tilde{\omega} = (g^3/\sigma)^{1/4}$ and $\kappa = (g/\sigma)^{1/2}$. In terms of K and Ω , the dispersion law (1.2) takes the form

$$\Omega^2 = K + K^3. \quad (3.6)$$

The non-dimensional spectrum of wave energy becomes

$$\tilde{S}(\Omega) = \tilde{Q}^{1/(\nu-1)} \frac{\Phi_\nu(\Omega)}{\nu-1} \left[1 + \frac{4(\nu-2)}{1+3K^2(\Omega)} \right] \Omega^{-\nu/(\nu-1)}, \quad (3.7)$$

and the non-dimensional spectrum of wave slope is

$$\tilde{F}_{\nabla\eta}(K) = \frac{K^{1/2}(1+3K^2)}{2(1+K^2)^{3/2}} \tilde{S}(\Omega(K)), \quad (3.8)$$

where $\tilde{F}_{\nabla\eta}(K) = (\kappa^2/\alpha) F_{\nabla\eta}(k)$.

4. Comparison with laboratory observations

To compare these results with the laboratory measurements by Jähne & Riemer (1990), we need the 'saturation function'

$$B(k) = k^2 F_{\nabla\eta}(k). \quad (4.1)$$

An example of the Jähne & Riemer measurements is reproduced in figure 1. The values of the energy flux can be expressed via the external parameters of the problem – the mean wind velocity, U , at a height, say, 10 m above the mean sea level:

$$Q = (\rho_a/\rho_w) R_q U^3, \quad (4.2)$$

where the density ratio is of order 10^{-3} and the bulk transfer coefficient of the wave energy, R_q , is somewhere between 3×10^{-5} and 4×10^{-5} – as follows from analysis of Miles' linear instability mechanism of wave generation (Zakharov 1992), from its empirical parameterizations (Glazman 1994), or from physical reasoning (V. E. Zakharov, 1994, personal communication): $R_q \approx (\rho_a/\rho_w)^{3/2}$.

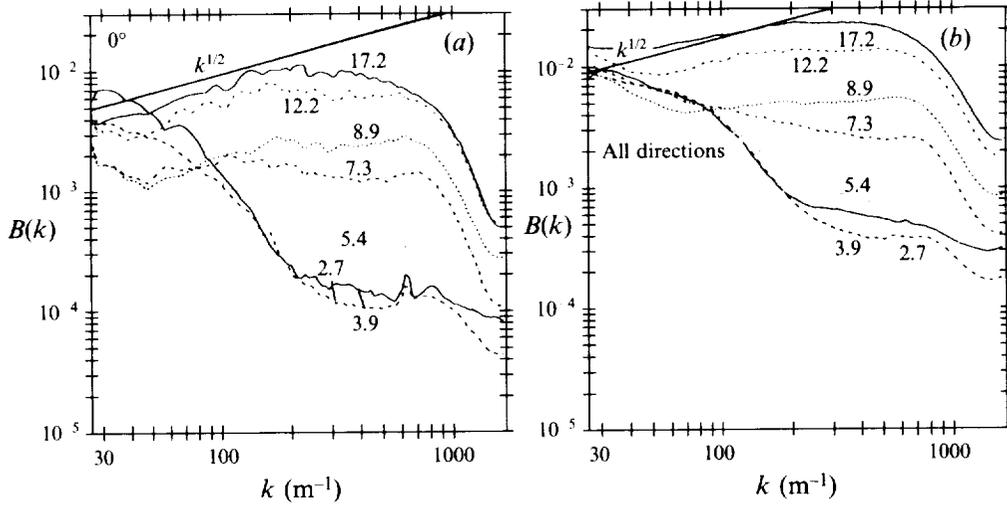


FIGURE 1. The saturation spectral function, $B(k)$, measured by Jähne & Riemer (1990) for wind speed values shown for each curve in m s^{-1} : (a) the linear section, $B(k_x)$ (in the along-wind direction), of the two-dimensional spectrum; (b) (Jähne & Riemer's (d)) the one-dimensional spectral function obtained by integrating $B(k)$ over all angles. (Courtesy of the authors.)

The $B(k)$ calculated in the weak turbulence limit is illustrated in figure 2 where the Kolmogorov constant, $\alpha = 2 \times 10^{-2}$, is chosen to provide the correct order of magnitude (as compared to the measured one) for the lowest curve with $U = 5.4 \text{ m s}^{-1}$. Apparently, the slow growth of the weak-turbulence spectrum with increasing wind is in strong disagreement with the results shown in figure 1. Moreover, the curves show no saturation at high wind. Finally, as follows from figure 1, the actual spectrum at low frequencies and high winds is close to the Phillips spectrum $B(k) = \text{const}$ rather than the Zakharov–Filonenko spectrum. All this indicates that the measured spectra are dominated by a rather high degree of wave nonlinearity. In order to obtain a better agreement with the observations, we shall now permit ν to grow as a function of increasing wind.

In principle, ν can be related to the energy flux Q arriving from the low-frequency range and to the magnitude of the wave spectrum in that range. Formally, this is done by matching spectrum (2.18) to the known spectrum of gravity waves at some characteristic frequency, ω_0 , chosen as a boundary between the ‘energy supply’ range and the inertial capillary–gravity range. Thus we employ the same sort of compatibility condition as used in the earlier study (Glazman 1992). For such a frequency, ω_0 (for which $k_0/\kappa \ll 1$), equation (2.18) simplifies to

$$S(\omega_0) = \alpha' \rho g^3 (\nu-2)/(\nu-1) Q^{1/(\nu-1)} \omega_0^{-(5\nu-8)/(\nu-1)}, \quad (4.3)$$

where $\alpha' = \alpha(4\nu-7)/(\nu-1)$. As ν varies from 4 to infinity, the ratio α'/α changes only from 3 to 4. This variation is negligible compared to variations in the other factors of (4.3). Demanding $S(\omega_0) = S_0$ where S_0 is considered to be known, we obtain an equation for ν . Neglecting the weak dependence of α' on ν the solution is found as

$$\nu = 1 + \frac{\ln(Q\omega_0^3/g^3)}{\ln(S_0\omega_0^5/\alpha'\rho g^3)}. \quad (4.4)$$

While this expression confirms that ν is an increasing function of wind, its practical use

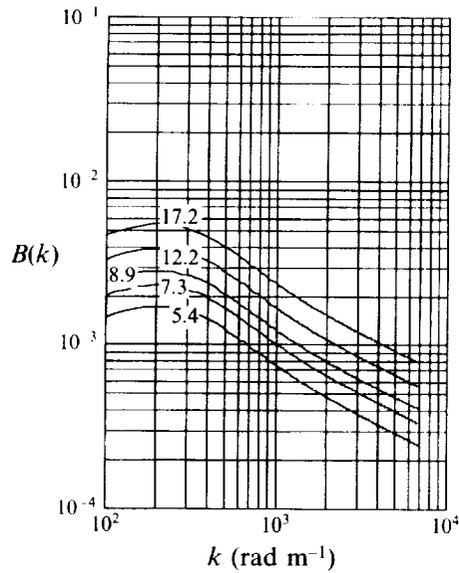


FIGURE 2. The theoretical saturation function, $B(k)$, for weakly nonlinear waves ($\nu = 4$). Here, $\sigma = 70 \text{ cm}^3 \text{ s}^{-2}$, $g = 981 \text{ cm s}^{-2}$, and $\alpha = 0.02$. Numbers at the curves show wind speed U in m s^{-1} .

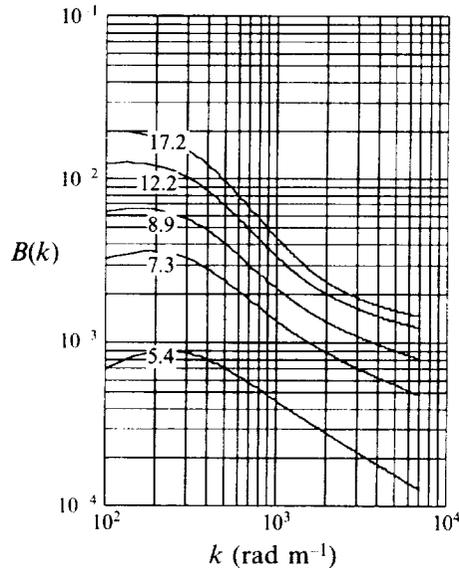


FIGURE 3. The theoretical saturation function, $B(k)$, for $\nu(U) = 0.2 + 0.6U$. Here, $\sigma = 70 \text{ cm}^3 \text{ s}^{-2}$, $g = 981 \text{ cm s}^{-2}$, and $\alpha = 0.02$. Numbers at the curves show wind speed U in m s^{-1} .

is limited because it requires knowledge of the wave spectrum at low frequencies. To provide S_0 , one would have to consider the entire wave-generation problem, a grand task well beyond the scope of the present work. Moreover, with respect to the present experimental comparison, such a formal determination of ν might be irrelevant. Indeed, laboratory experiments greatly limit the wave age by inhibiting the development of the inertial cascade in deep-water gravity waves (due to a short wind

fetch and limited water depth) and thus creating in that range an artificial physical situation. Hence, the determination of S_0 would present a special problem.

An alternative (and more instructive) approach is to fit some trial function $\nu(U)$ which is reasonable from the physical standpoint. Using, for instance, the simplest linear function $\nu = a + bU$ one can determine empirical coefficients a and b providing the best agreement between the theoretical and experimental spectra. Coefficient b will give us an idea about the rate at which ν grows with increasing energy input. This empirical procedure yielded $a = 0.2$ and $b = 0.6 \text{ (m s}^{-1}\text{)}^{-1}$ which corresponds to $\nu = 3.2$ for $U = 5 \text{ m s}^{-1}$ and $\nu = 9.2$ for $U = 15 \text{ m s}^{-1}$; the resulting spectra are plotted in figure 3. The Kolmogorov constant used in figure 3, $\alpha = 2 \times 10^{-2}$, is consistent with an earlier determination (figures 1 and 3 in Glazman 1992). Apparently, a monotonic growth of ν with increasing wind leads to a better agreement with the experiment. Assuming a more complicated function $\nu(U)$ this agreement can be improved further.

Evidently, the present theory successfully explains several prominent features observed in the experiment: the gradually diminishing growth of the spectral level with increasing wind, an accelerated roll-off in the transitional range between the gravity and the capillary regimes, and the approximate independence of the saturation function, B , of k in the low-wavenumber end of our range.

The main quantitative discrepancy between the predicted and the measured spectrum is that the accelerated roll-off of the saturation function observed in the experiment commences at wavenumbers about three times as high as those predicted in figures 2 and 3. This discrepancy appears to be due to the possible influence of low-frequency waves present in actual experiments. The long waves would cause a Doppler shift of the capillary-gravity wave frequency and would also lead to a non-local energy exchange between the small-scale ripples and the long waves – discussed in §1.

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